Analytical Study of Non-Universality of the Soft Terms in the MSSM

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Abstract

We obtain general analytical forms for the solutions of the one-loop renormalization group equations in the top/bottom/τ sector of the MSSM. These solutions are valid for any value of $\tan\beta$ as well as any non-universal initial conditions for the soft SUSY breaking parameters and non-unification of the Yukawa couplings. We establish analytically a generic screening effect of non-universality, in the vicinity of the infrared quasi fixed point, which allows to determine sector-wise a hierarchy of sensitivity to initial conditions. We give also various numerical illustrations of this effect away from the quasi fixed point and assess the sensitivity of the Higgs and sfermion spectra to the non-universality of the various soft breaking sectors. As a by-product, a typical anomaly-mediated non-universality of the gaugino sector would have marginal influence on the scalar spectrum.

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1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) [1] has been intensively studied recently [2] with the emphasis on prediction of particle spectrum. It crucially depends on the mechanism of SUSY breaking and the one which is commonly accepted introduces the so-called soft terms at a high energy scale [3, 4]. These soft terms are running then down to low energy according to the well known RG equations starting from some initial values. In the minimal version the soft terms obey the universality hypothesis which leaves one with a set of 5 independent parameters (before radiative electroweak symmetry breaking) : $m_0, m_{1/2}, A_0, \mu_0$ and $\tan \beta$. However, recently there appeared some interest in new contributions to SUSY breaking patterns which result in non-universal boundary conditions for the soft terms [5]. To investigate the influence of non-universality on the particle spectrum it is very useful to have analytical solutions to the RG equations for the evolution of the soft terms. Numerical analysis though straightforward is rather complicated due to a large number of free parameters while an analytical solution allows one to see which of these parameters is important and which is not.

While with a single Yukawa coupling the analytical solution to the one-loop RG equations has been known for long, for increasing number of Yukawa couplings it has been obtained quite recently [6] in the form that allows iterative presentation. On the other hand, it has been shown [7] that when one knows the solution to RG equations for the couplings of a rigid theory, one can obtain those for the soft terms by a usual Taylor expansion over the Grassmannian parameters. Thus one can apply the Grassmannian expansion to the general solutions of [6] to get the analytical solution for the soft terms with arbitrary initial conditions in an iterative form and explore their dependence on the latter.

In the present paper we perform this analysis in the MSSM with three Yukawa couplings $(Y_t, Y_b$ and $Y_\tau)$ and arbitrary initial conditions for the soft terms in the one-loop approximation. We examine the dependence of the solutions on the initial conditions and show that if the Yukawa couplings are large enough (typically bigger than α_s at the GUT scale) the solutions exhibit a quasi-fixed point behaviour. This means that in such a regime some of the initial conditions are completely washed out, and become actually inessential at low energy. The main role is played by the gaugino masses out of which the gluino mass is by far dominating.

2 Analytic Solution to the RG Equations for Yukawa Couplings

Though the RG equations for the Yukawa couplings do not have explicit analytic solution, they can be solved iteratively as it has been demonstrated in Ref.[6]. Using the notation

$$\alpha_i = \frac{g_i^2}{16\pi^2}, \ i = 1, 2, 3; \quad Y_k = \frac{y_k^2}{16\pi^2}, \ k = t, b, \tau$$

one can write down the one-loop RG equations as

$$\dot{\alpha}_i = -b_i \alpha_i^2, \tag{1}$$

$$\dot{Y}_k = Y_k (\sum_i c_{ki}\alpha_i - \sum_l a_{kl}Y_l), \tag{2}$$

where $\cdot \equiv d/dt$, $t = \log M_{GUT}^2/Q^2$ and

$$b_i = \{33/5, 1, -3\},$$
$$c_{ti} = \{13/15, 3, 16/3\}, \quad c_{bi} = \{7/15, 3, 16/3\}, \quad c_{\tau i} = \{9/5, 3, 0\},$$
$$a_{tl} = \{6, 1, 0\}, \quad a_{bl} = \{1, 6, 1\}, \quad a_{\tau l} = \{0, 3, 4\}.$$

The general solution to eqs.(1,2) can be written as

$$\alpha_i = \frac{\alpha_i^0}{1 + b_i \alpha_i^0 t}, \tag{3}$$

$$Y_k = \frac{Y_k^0 u_k}{1 + a_{kk} Y_k^0 \int_0^t u_k}, \tag{4}$$

where the functions u_k obey the integral system of equations

$$u_t = \frac{E_t}{(1 + 6Y_b^0 \int_0^t u_b)^{1/6}}, \quad u_b = \frac{E_b}{(1 + 6Y_t^0 \int_0^t u_t)^{1/6}(1 + 4Y_\tau^0 \int_0^t u_\tau)^{1/4}}, \quad u_\tau = \frac{E_\tau}{(1 + 6Y_b^0 \int_0^t u_b)^{1/2}}, \tag{5}$$

and the functions E_k are given by

$$E_k = \prod_{i=1}^{3}(1 + b_i \alpha_i^0 t)^{c_{ki}/b_i}. \tag{6}$$

Let us stress that eqs.(3,4) give the exact solution to eqs.(1,2), while the u_k's in eqs.(5), although solved formally in terms of the E_k's and Y_k^0's as continued integrated fractions, should in practice be solved iteratively. Yet the important gain here is twofold:

i) as shown in [6], the convergence of the successive iterations to the exact solution can be fully controlled analytically in terms of the initial values of the Yukawas, allowing in practice to obtain approximations at the level of the percent or less after one or two iterations and

ii) the structure of the solutions is explicit enough to allow for exact statements about some regimes of the initial conditions, as we will see later on. Furthermore, these nice features will be naturally passed on to the solutions for the soft SUSY breaking parameters since the latter will be obtained from (3–5) through the method of Ref.[7].

3   The Soft Terms and Grassmannian Expansion

An important feature of the solution (4,5) is that it is written in an analytic form with the initial conditions explicitly present. This allows one to get the same type of solution for all the soft terms in an iterative form.

Let us describe briefly the procedure. It has been shown[8] that the soft terms which break supersymmetry can be introduced in a classical Lagrangian via the so-called spurion fields. This leads to the modification of the original couplings of a rigid theory, they become external spurion superfields depending on Grassmannian parameters [9][1]. In the MSSM it looks like

$$\alpha \to \tilde{\alpha}_i \;=\; \alpha_i(1 + m_i\eta + \bar{m}_i\bar{\eta} + 2m_i\bar{m}_i\eta\bar{\eta}), \tag{7}$$

$$Y_k \to \tilde{Y}_k \;=\; Y_k(1 - A_k\eta - \bar{A}_k\bar{\eta} + (\Sigma_k + A_k\bar{A}_k)\eta\bar{\eta}), \tag{8}$$

where m_i are the gaugino masses, A_k are the scalar triple couplings and Σ_k are certain combinations of the soft masses

$$\Sigma_t = \tilde{m}_{Q3}^2 + \tilde{m}_{U3}^2 + m_{H2}^2, \quad \Sigma_b = \tilde{m}_{Q3}^2 + \tilde{m}_{D3}^2 + m_{H1}^2, \quad \Sigma_\tau = \tilde{m}_{L3}^2 + \tilde{m}_{E3}^2 + m_{H1}^2.$$

Here $\eta = \theta^2$ and $\bar{\eta} = \bar{\theta}^2$ are the spurion fields depending on Grassmannian parameters $\theta_\alpha, \bar{\theta}_\alpha$ ($\alpha = 1, 2$).

It has been proven in Ref.[9] that the singular part of effective action, which determines the renormalization properties of any softly broken SUSY theory, is equal to that of an unbroken one in presence of external spurion superfields. This means that in order to calculate it in a softly broken case one just has to take the unbroken one, replace the couplings according to eqs.(7,8) and expand over the Grassmannian parameters η and $\bar{\eta}$.

Moreover, as it has been demonstrated in [7], the same replacement can be done directly in RG equations in order to get the corresponding equations for the soft terms, or even in the solutions to these equations. In the last case one obtains the solutions to the RG equations for the soft terms. Below we demonstrate how this procedure works in case of the MSSM, when combined with the solutions (3,4).

4   Analytical Solution to RG Equations for the Soft Terms

Let us now perform the substitution (7,8) in (3-5) and expand over η and $\bar{\eta}$. Then the linear term in η will give us the solution for m_i and A_k and the $\eta\bar{\eta}$ terms the ones for Σ_k. (For simplicity, we do not consider here CP-violating effects and take all the soft parameters to be real valued.) The resulting exact solutions look similar to those for the rigid couplings (3–5)

$$m_i \;=\; \frac{m_i^0}{1 + b_i\alpha_i^0 t}, \tag{9}$$

$$A_k \;=\; -e_k + \frac{A_k^0/Y_k^0 + a_{kk}\int u_k e_k}{1/Y_k^0 + a_{kk}\int u_k}, \tag{10}$$

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[1]The resulting formulae coincide with those of Ref.[10] except for some minor difference in higher loops. Since we consider only one-loop RG equations we ignore this difference here. Similar results were obtained also in Ref.[11]

$$\Sigma_k = \xi_k + A_k^2 + 2e_k A_k - \frac{(A_k^0)^2/Y_k^0 - \Sigma_k^0/Y_k^0 + a_{kk}\int u_k\xi_k}{1/Y_k^0 + a_{kk}\int u_k}, \tag{11}$$

where the new functions e_k and ξ_k have been introduced which obey the iteration equations

$$e_t = \frac{1}{E_t}\frac{d\tilde{E}_t}{d\eta} + \frac{A_b^0\int u_b - \int u_b e_b}{1/Y_b^0 + 6\int u_b},$$

$$e_b = \frac{1}{E_b}\frac{d\tilde{E}_b}{d\eta} + \frac{A_t^0\int u_t - \int u_t e_t}{1/Y_t^0 + 6\int u_t} + \frac{A_\tau^0\int u_\tau - \int u_\tau e_\tau}{1/Y_\tau^0 + 4\int u_\tau},$$

$$e_\tau = \frac{1}{E_\tau}\frac{d\tilde{E}_\tau}{d\eta} + 3\frac{A_b^0\int u_b - \int u_b e_b}{1/Y_b^0 + 6\int u_b},$$

$$\xi_t = \frac{1}{E_t}\frac{d^2\tilde{E}_t}{d\eta d\bar{\eta}} + 2\frac{1}{E_t}\frac{d\tilde{E}_t}{d\eta}\frac{A_b^0\int u_b - \int u_b e_b}{1/Y_b^0 + 6\int u_b} + 7\left(\frac{A_b^0\int u_b - \int u_b e_b}{1/Y_b^0 + 6\int u_b}\right)^2$$
$$- \left((\Sigma_b^0 + (A_b^0)^2)\int u_b - 2A_b^0\int u_b e_b + \int u_b\xi_b\right)/\left(\frac{1}{Y_b^0} + 6\int u_b\right),$$

$$\xi_b = \frac{1}{E_b}\frac{d^2\tilde{E}_b}{d\eta d\bar{\eta}} + 2\frac{1}{E_b}\frac{d\tilde{E}_b}{d\eta}\left[\frac{A_t^0\int u_t - \int u_t e_t}{1/Y_t^0 + 6\int u_t} + \frac{A_\tau^0\int u_\tau - \int u_\tau e_\tau}{1/Y_\tau^0 + 4\int u_\tau}\right]$$
$$+7\left(\frac{A_t^0\int u_t - \int u_t e_t}{1/Y_t^0 + 6\int u_t}\right)^2 + 5\left(\frac{A_\tau^0\int u_\tau - \int u_\tau e_\tau}{1/Y_\tau^0 + 4\int u_\tau}\right)^2$$
$$+2\left(\frac{A_t^0\int u_t - \int u_t e_t}{1/Y_t^0 + 6\int u_t}\right)\left(\frac{A_\tau^0\int u_\tau - \int u_\tau e_\tau}{1/Y_\tau^0 + 4\int u_\tau}\right)$$
$$-\left((\Sigma_t^0 + (A_t^0)^2)\int u_t - 2A_t^0\int u_t e_t + \int u_t\xi_t\right)/\left(\frac{1}{Y_t^0} + 6\int u_t\right)$$
$$-\left((\Sigma_\tau^0 + (A_\tau^0)^2)\int u_\tau - 2A_\tau^0\int u_\tau e_\tau + \int u_\tau\xi_\tau\right)/\left(\frac{1}{Y_\tau^0} + 4\int u_\tau\right),$$

$$\xi_\tau = \frac{1}{E_\tau}\frac{d^2\tilde{E}_\tau}{d\eta d\bar{\eta}} + 6\frac{1}{E_\tau}\frac{d\tilde{E}_\tau}{d\eta}\frac{A_b^0\int u_b - \int u_b e_b}{1/Y_b^0 + 6\int u_b} + 27\left(\frac{A_b^0\int u_b - \int u_b e_b}{1/Y_b^0 + 6\int u_b}\right)^2$$
$$-3\left((\Sigma_b^0 + (A_b^0)^2)\int u_b - 2A_b^0\int u_b e_b + \int u_b\xi_b\right)/\left(\frac{1}{Y_b^0} + 6\int u_b\right). \tag{12}$$

where the variations of $\tilde{E}_k$ should be taken at $\eta = \bar{\eta} = 0$ and are given by

$$\frac{1}{E_k}\frac{d\tilde{E}_k}{d\eta}\bigg|_{\eta,\bar{\eta}=0} = t\sum_{i=1}^{3} c_{ki}\alpha_i m_i^0, \tag{13}$$

$$\frac{1}{E_k}\frac{d^2\tilde{E}_k}{d\eta d\bar{\eta}}\bigg|_{\eta,\bar{\eta}=0} = t^2\left(\sum_{i=1}^{3} c_{ki}\alpha_i m_i^0\right)^2 + 2t\sum_{i=1}^{3} c_{ki}\alpha_i(m_i^0)^2 - t^2\sum_{i=1}^{3} c_{ki}b_i\alpha_i^2(m_i^0)^2. \tag{14}$$

When solving eqs.(5) and (12) in the n-th iteration one has to substitute in the r.h.s. the $(n-1)$-th iterative solution for all the corresponding functions.

5

In the particular case where $Y_b = Y_\tau = 0$ eqs.(5-12) give an exact and well known solutions already in the first iteration.

Let us finally note that upon inspection of the solutions (9–12), the A_i's and Σ_i's depend respectively linearly and quadratically on the initial conditions, as expected, and thus can be generally cast in the form:

$$A_{t,b,\tau}(t) = \sum_{j=t,b,\tau} a_j(t) A_j^0 + \sum_{k=1,2,3} b_k(t) m_k^0, \tag{15}$$

$$\Sigma_{t,b,\tau}(t) = \sum_{i,j=1,2,3} c_{ij}(t) m_i^0 m_j^0 + \sum_{i,j=t,b,\tau} d_{ij}(t) A_i^0 A_j^0 + \sum_{i=t,b,\tau,j=1,2,3} e_{ij}(t) A_i^0 m_j^0$$

$$+ k_t(t) \Sigma_t^0 + k_b(t) \Sigma_b^0 + k_\tau(t) \Sigma_\tau^0, \tag{16}$$

where the various running coefficients $a_j, b_k, c_{ij}, d_{ij}, e_{ij}$ and k_i are fully determined by our solutions and can be seen to depend exclusively on the initial conditions of the gauge and Yukawa couplings. In Sec.6 we will evaluate these coefficients at the E.W. scale, using a truncation of the general solutions.

5 Quasi-Fixed Points and the Independence of Initial Conditions

The solutions (3–5, 9–12) enjoy the nice property of exhibiting the explicit dependence on initial conditions and one can trace this dependence all the way down to the final results. This is of special importance for the non-universal case since one can see which of the parameters is essential and which is washed out during the evolution. In particular the solution for the Yukawa couplings exhibit the fixed point behaviour when the initial values are large enough. More precisely, in the regime $Y_t^0, Y_b^0, Y_\tau^0 \to \infty$ with fixed finite ratios $Y_t^0/Y_b^0 = r_1, Y_b^0/Y_\tau^0 = r_2$, it is legitimate to drop 1 in the denominators of eqs.(4, 5) (see appendix A for a proof) in which case the exact Yukawa solutions go to the so-called IR quasi-fixed points (IRQFP) defined by

$$Y_k^{FP} = \frac{u_k^{FP}}{a_{kk} \int u_k^{FP}} \tag{17}$$

with

$$u_t^{FP} = \frac{E_t}{(\int u_b^{FP})^{1/6}}, \quad u_b^{FP} = \frac{E_b}{(\int u_t^{FP})^{1/6}(\int u_\tau^{FP})^{1/4}}, \quad u_\tau^{FP} = \frac{E_\tau}{(\int u_b^{FP})^{1/2}} \tag{18}$$

extending the IRQFP [12] to three Yukawa couplings. What is worth stressing here is that both the dependence on the initial condition for each Yukawa as well as the effect of Yukawa non-unification, r_1, r_2 have completely dropped out of the runnings. (Note that in practice this regime is already obtained if $Y_k^0 \geq \alpha_0^{GUT}$, assuming here for simplicity the unification of the gauge couplings $\alpha_1^0 = \alpha_2^0 = \alpha_3^0 = \alpha_0^{GUT}$.) The fact that the ratios r_1, r_2 drop out implies the validity of the described properties in any $\tan \beta$ regime.

This in turn leads to the IRQFPs for the soft terms. Disappearance of Y_k^0 in the FP solution naturally leads to the disappearance of A_k^0 and Σ_k^0 in the soft term fixed points. To see this one can either go to the limit of large Y_k^0 in eqs.(9–12) or directly perform the Grassmannian expansion of the FP solutions (17,18). One gets

$$A_k^{FP} = -e_k^{FP} + \frac{\int u_k^{FP} e_k^{FP}}{\int u_k^{FP}}, \tag{19}$$

$$\Sigma_k^{FP} = \xi_k^{FP} + (A_k^{FP})^2 + 2e_k^{FP} A_k^{FP} - \frac{\int u_k^{FP} \xi_k^{FP}}{\int u_k^{FP}}$$

$$= \xi_k^{FP} - \frac{\int u_k^{FP} \xi_k^{FP}}{\int u_k^{FP}} - (e_k^{FP})^2 + \left(\frac{\int u_k^{FP} e_k^{FP}}{\int u_k^{FP}}\right)^2 \tag{20}$$

with

$$e_t^{FP} = \frac{1}{E_t}\frac{d\tilde{E}_t}{d\eta} - \frac{1}{6}\frac{\int u_b^{FP} e_b^{FP}}{\int u_b^{FP}},$$

$$e_b^{FP} = \frac{1}{E_b}\frac{d\tilde{E}_b}{d\eta} - \frac{1}{6}\frac{\int u_t^{FP} e_t^{FP}}{\int u_t^{FP}} - \frac{1}{4}\frac{\int u_\tau^{FP} e_\tau^{FP}}{\int u_\tau^{FP}},$$

$$e_\tau^{FP} = \frac{1}{E_\tau}\frac{d\tilde{E}_\tau}{d\eta} - \frac{1}{2}\frac{\int u_b^{FP} e_b^{FP}}{\int u_b^{FP}},$$

$$\tag{21}$$

$$\xi_t^{FP} = \frac{1}{E_t}\frac{d^2\tilde{E}_t}{d\eta d\bar\eta} + \frac{7}{36}\left(\frac{\int u_b^{FP} e_b^{FP}}{\int u_b^{FP}}\right)^2 - \frac{1}{3}\frac{1}{E_t}\frac{d\tilde{E}_t}{d\eta}\frac{\int u_b^{FP} e_b^{FP}}{\int u_b^{FP}} - \frac{1}{6}\frac{\int u_b^{FP} \xi_b^{FP}}{\int u_b^{FP}},$$

$$\xi_b^{FP} = \frac{1}{E_b}\frac{d^2\tilde{E}_b}{d\eta d\bar\eta} + \frac{7}{36}\left(\frac{\int u_t^{FP} e_t^{FP}}{\int u_t^{FP}}\right)^2 + \frac{5}{16}\left(\frac{\int u_\tau^{FP} e_\tau^{FP}}{\int u_\tau^{FP}}\right)^2$$

$$- \frac{1}{E_b}\frac{d\tilde{E}_b}{d\eta}\left(\frac{1}{3}\frac{\int u_t^{FP} e_t^{FP}}{\int u_t^{FP}} + \frac{1}{2}\frac{\int u_\tau^{FP} e_\tau^{FP}}{\int u_\tau^{FP}}\right) - \frac{1}{6}\frac{\int u_t^{FP} \xi_t^{FP}}{\int u_t^{FP}} - \frac{1}{4}\frac{\int u_\tau^{FP} \xi_\tau^{FP}}{\int u_\tau^{FP}},$$

$$\xi_\tau^{FP} = \frac{1}{E_\tau}\frac{d^2\tilde{E}_\tau}{d\eta d\bar\eta} + \frac{3}{4}\left(\frac{\int u_b^{FP} e_b^{FP}}{\int u_b^{FP}}\right)^2 - \frac{1}{E_\tau}\frac{d\tilde{E}_\tau}{d\eta}\frac{\int u_b^{FP} e_b^{FP}}{\int u_b^{FP}} - \frac{1}{2}\frac{\int u_b^{FP} \xi_b^{FP}}{\int u_b^{FP}}.$$

One can see from eqs.(19,20) that the constant terms in e_k and ξ_k do not contribute to A_k and Σ_k and can be dropped from eqs.(21). Thus, all the dependence on the initial conditions Y_k^0, A_k^0 and Σ_k^0 disappears from the fixed point solutions. The only dependence left is on the gaugino masses. This is a general screening property valid for the exact solution as well as for any of its approximate (truncated) forms.

In view of the above screening properties of the initial conditions at the quasi-fixed point, one should recall the existing connection between the true IR attractive fixed point of the Yukawa couplings and that of the soft parameters [13]. What we have established can be seen as an extension of such connections to the transient regime of quasi-fixed point at the one-loop level. It is also worth stressing that the above properties are valid for any renormalization scale and are thus operative despite the uncertainty in the choice of the physical scales.

6   Numerical Analysis. The Role of Non-universality

We perform now a numerical study of our solutions in the first and next iterations in order to demonstrate the convergence of the procedure and to show the role of certain initial conditions. In particular we will show that assuming that the soft terms are of the same order of magnitude at the GUT scale the only essential parameter is the gluino mass. All the other soft terms are suppressed either due to the fixed point behaviour mentioned above, or due to the smallness of α_1 and α_2 compared to α_3 at low energy.

6.1   The first iteration

To have a solution in the first iteration one takes eqs.(5,12) where in the r.h.s. the functions u_k, e_k and ξ_k are taken in the 0-th iteration. They are

$$u_k = E_k, \quad e_k = \frac{1}{E_k}\frac{d\tilde{E}_k}{d\eta}, \quad \xi_k = \frac{1}{E_k}\frac{d^2\tilde{E}_k}{d\eta d\bar{\eta}}, \tag{22}$$

where the variations of E_k are given by eqs.(13,14).

For comparison we consider two particular cases:

I) $Y_t^0 = 5\alpha_0, \quad Y_b^0 = Y_\tau^0 = 0,$

II) $Y_t^0 = Y_b^0 = Y_\tau^0 = (1 \div 10)\alpha_0,$ (with $\alpha_0 \approx 0.00329$).

The first case corresponds to the so-called low $\tan\beta$ regime, where the first iteration is already exact, and the second case refers to the high $\tan\beta$ regime for an $SO(10)$-like initial conditions.

In the first case the only essential soft terms besides the gaugino masses are A_t and Σ_t. All the soft masses of the third generation are expressed through Σ_t in a simple way[14]. A_t and Σ_t have a dimension of a mass and mass squared, respectively, and may be expanded over the initial conditions in the following fashion (eqs.(15, 16),

$$\begin{aligned} A_t &= 0.0359\, A_{t0} - 0.0167\, m_{10} - 0.1598\, m_{20} - 1.6284\, m_{30}, \tag{23}\\ \Sigma_t &= -0.0346\, A_{t0}^2 + 0.0044\, A_{t0}m_{10} + 0.0270\, A_{t0}m_{20} + 0.1225\, A_{t0}m_{30}\\ &\quad +0.0184\, m_{10}^2 - 0.0057\, m_{10}m_{20} + 0.2566\, m_{20}^2\\ &\quad -0.0335\, m_{10}m_{30} - 0.2735\, m_{20}m_{30} + 6.3695\, m_{30}^2\\ &\quad +0.0359\, (m_{H_2}^0)^2 + 0.0359\, (\tilde{m}_{Q_3}^0)^2 + 0.0359\, (\tilde{m}_{U_3}^0)^2, \tag{24} \end{aligned}$$

where the numbers are calculated for $t = \log M_{GUT}^2/M_Z^2 \approx 66$.

One can see from eqs.(23,24) that the prevailing term is that of m_3^0. A_t^0 and Σ_t^0 decouple due to the fixed point behaviour as explained above and the contribution of m_1^0 and m_2^0 is small due to smallness of α_1 and α_2 compared to α_3 at $t = 66$.

The second case looks similar. We first consider the triple couplings A_k. In this case one has a set of initial values $\{A_k^0, m_i^0\}$. In fig.1 we show the variation of the coefficients a_i, b_k of eq.(15) for A_t, A_b and A_τ as functions of Y_k^0 in the interval $Y_k^0 = \alpha_0 \div 10\alpha_0$

One can clearly see that the coefficients of A_k^0 are small and have a fast decrease with increasing Y_k^0. The coefficients of m_i^0 quickly saturate and approach their asymptotic

values with the hierarchy $1:10:100$ for m_1^0, m_2^0 and m_3^0. The effect is less pronounced for A_τ due to the absence of the SU(3) coupling in the lepton sector.

Now we come to Σ_k. We have chosen the intermediate value of $Y_k^0 = 5\alpha_0$ where the effective fixed point is practically already reached and calculate the coefficients at $t = 66$ as in eqs.(23,24). One has

$$
\begin{aligned}
\Sigma_t =\ & -0.0497\ A_{t0}^2 + 0.0076\ A_{b0}^2 + 0.0142\ A_{t0}A_{b0} + 0.0057\ A_{t0}m_{10} \\
& -0.0018\ A_{b0}m_{10} + 0.0333\ A_{t0}m_{20} - 0.0114\ A_{b0}m_{20} + 0.1509\ A_{t0}m_{30} \\
& -0.0516\ A_{b0}m_{30} + 0.0198\ m_{10}^2 + 0.2509\ m_{20}^2 + 6.3299\ m_{30}^2 \\
& -0.0057\ m_{10}m_{20} - 0.0336\ m_{10}m_{30} - 0.2669\ m_{20}m_{30} - 0.0252\ (\tilde{m}_{D_3}^0)^2 \\
& -0.0252\ (m_{H_1}^0)^2 + 0.0525\ (m_{H_2}^0)^2 + 0.0273\ (\tilde{m}_{Q_3}^0)^2 + 0.0525\ (\tilde{m}_{U_3}^0)^2, \quad (25)
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_b =\ & +0.0079\ A_{t0}^2 - 0.0717\ A_{b0}^2 + 0.0058\ A_{\tau0}^2 + 0.0200\ A_{t0}A_{b0} \\
& -0.0062\ A_{t0}A_{\tau0} + 0.0262\ A_{b0}A_{\tau0} - 0.0005\ A_{t0}m_{10} + 0.0013\ A_{b0}m_{10} \\
& +0.0004\ A_{\tau0}m_{10} - 0.0119\ A_{t0}m_{20} + 0.0395\ A_{b0}m_{20} - 0.0121\ A_{\tau0}m_{20} \\
& -0.0618\ A_{t0}m_{30} + 0.2008\ A_{b0}m_{30} - 0.0965\ A_{\tau0}m_{30} - 0.0052\ m_{10}^2 \\
& +0.2359\ m_{20}^2 + 6.8165\ m_{30}^2 - 0.0027\ m_{10}m_{20} - 0.0037\ m_{10}m_{30} \\
& -0.2498\ m_{20}m_{30} + 0.0778\ (\tilde{m}_{D_3}^0)^2 - 0.0502\ (\tilde{m}_{E_3}^0)^2 + 0.0276\ (m_{H_1}^0)^2 \\
& -0.0324\ (m_{H_2}^0)^2 - 0.0502\ (\tilde{m}_{L_3}^0)^2 + 0.0454\ (\tilde{m}_{Q_3}^0)^2 - 0.0325\ (\tilde{m}_{U_3}^0)^2, \quad (26)
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_\tau =\ & -0.0010\ A_{b0}^2 - 0.1947\ A_{\tau0}^2 + 0.1397\ A_{b0}A_{\tau0} - 0.0195\ A_{b0}m_{10} \\
& +0.0339\ A_{\tau0}m_{10} - 0.0409\ A_{b0}m_{20} + 0.0718\ A_{\tau0}m_{20} + 0.0894\ A_{b0}m_{30} \\
& -0.0842\ A_{\tau0}m_{30} + 0.0993m_{10}^2 + 0.3527\ m_{20}^2 - 2.8162\ m_{30}^2 \\
& -0.0133\ m_{10}m_{20} + 0.0116\ m_{10}m_{30} - 0.0687\ m_{20}m_{30} - 0.2177\ (\tilde{m}_{D_3}^0)^2 \\
& +0.2649\ (\tilde{m}_{E_3}^0)^2 + 0.0473\ (m_{H_1}^0)^2 + 0.2649\ (\tilde{m}_{L_3}^0)^2 - 0.2177\ (\tilde{m}_{Q_3}^0)^2. \quad (27)
\end{aligned}
$$

One can again see how the coefficients of the initial values of A_k^0 and Σ_k^0 almost vanish and the prevailing one is that of $(m_3^0)^2$. The next-to-leading ones are those of $(m_2^0)^2$ and $m_2^0 m_3^0$ being however almost 30 times smaller. This is true for both Σ_t^0 and Σ_b^0 but is less manifest for Σ_τ^0. We note here that a soft gaugino mass hierarchy like the one predicted by anomaly-mediated susy breaking, $m_3^0 : m_2^0 : m_1^0 = 3 : 0.3 : 1$ [5], enforces even more the insensitivity of the running A_i's and Σ_i's to the non-universality of the gaugino sector.

6.2   The next iterations

To demonstrate the validity of the iterative procedure and reliability of the the first iteration we consider the effect of the next ones on the above mentioned coefficients. We have performed the numerical integration up to the 6-th iteration and have observed fast convergence of the coefficients to their exact values. To show the numbers we have chosen the leading coefficients of m_{03} in A_k and m_{03}^2 in Σ_k. In case $Y_t^0 = Y_b^0 = Y_\tau^0 = 5\alpha_0$ and

$\alpha_0 = 0.00329$ the results are the following:

Iteration	A_t	A_b	A_τ	Σ_t	Σ_b	Σ_τ
1st	-1.6127	-1.7584	0.6871	6.3299	6.8166	-2.8162
2nd	-1.6161	-1.7330	0.5037	6.3270	6.6822	-2.0989
3rd	-1.6125	-1.7372	0.5526	6.3192	6.7069	-2.2937
4th	-1.6133	-1.7375	0.5440	6.3213	6.7054	-2.2588
5th	-1.6131	-1.7373	0.5456	6.3206	6.7053	-2.2660
6th	-1.6131	-1.7374	0.5454	6.3207	6.7054	-2.2649

One can see explicitly the fast convergence of the iterations. As expected it is worse for A_τ and Σ_τ, so in this case one has to take few more iterations. We present the general arguments for the convergence of iterations for the soft terms in appendix B. The advantage of this solution is that one can improve the precision taking further iterations and in principle can achieve any desirable accuracy. Typically one has an accuracy of a few percent after 2-3 iterations. This is in contrast with the approximate solutions presented in Ref.[14] which give simple explicit expressions but without improvement.

Taking the sixth iteration in eqs.(5,12) expressions for the soft terms now look like

$$A_t = 0.0558\, A_{t0} - 0.0294\, A_{b0} + 0.0080\, A_{\tau0} - 0.0186\, m_{10} - 0.1586\, m_{20} - 1.6131\, m_{30},$$
$$A_b = -0.0341\, A_{t0} + 0.0984\, A_{b0} - 0.0450\, A_{\tau0} - 0.0014\, m_{10} - 0.1583\, m_{20} - 1.7374\, m_{30},$$
$$A_\tau = 0.0394\, A_{t0} - 0.2221\, A_{b0} + 0.2871\, A_{\tau0} - 0.0825\, m_{10} - 0.2344\, m_{20} + 0.5454\, m_{30},$$

$$\begin{aligned}
\Sigma_t = {}& -0.0487\, A_{t0}^2 + 0.0068\, A_{b0}^2 + 0.0014\, A_{\tau0}^2 + 0.0130\, A_{t0}A_{b0} \\
& -0.0017\, A_{t0}A_{\tau0} - 0.0017\, A_{b0}A_{\tau0} + 0.0058\, A_{t0}m_{10} - 0.0015\, A_{b0}m_{10} \\
& -0.0001\, A_{\tau0}m_{10} + 0.0335\, A_{t0}m_{20} - 0.0097\, A_{b0}m_{20} + 0.0005\, A_{\tau0}m_{20} \\
& +0.1514\, A_{t0}m_{30} - 0.0460\, A_{b0}m_{30} + 0.0070\, A_{\tau0}m_{30} + 0.0211\, m_{10}^2 \\
& +0.2547\, m_{20}^2 + 6.3207\, m_{30}^2 - 0.0057\, m_{10}m_{20} - 0.0340\, m_{10}m_{30} \\
& -0.2720\, m_{20}m_{30} - 0.0294\, (\tilde{m}_{D_3}^0)^2 - 0.0214\, (m_{H_1}^0)^2 + 0.0558\, (m_{H_2}^0)^2 \\
& +0.0264\, (\tilde{m}_{Q_3}^0)^2 + 0.0558\, (\tilde{m}_{U_3}^0)^2 + 0.0080\, (\tilde{m}_{L_3}^0)^2 + 0.0080\, (\tilde{m}_{E_3}^0)^2,
\end{aligned}$$

$$\begin{aligned}
\Sigma_b = {}& +0.0067\, A_{t0}^2 - 0.0736\, A_{b0}^2 - 0.0035\, A_{\tau0}^2 + 0.0200\, A_{t0}A_{b0} \\
& -0.0064\, A_{t0}A_{\tau0} + 0.0302\, A_{b0}A_{\tau0} - 0.0003\, A_{t0}m_{10} - 0.0000\, A_{b0}m_{10} \\
& +0.0021\, A_{\tau0}m_{10} - 0.0110\, A_{t0}m_{20} + 0.0396\, A_{b0}m_{20} - 0.0076\, A_{\tau0}m_{20} \\
& -0.0613\, A_{t0}m_{30} + 0.2356\, A_{b0}m_{30} - 0.0945\, A_{\tau0}m_{30} - 0.0013\, m_{10}^2 \\
& +0.2532\, m_{20}^2 + 6.7053\, m_{30}^2 - 0.0030\, m_{10}m_{20} - 0.0053\, m_{10}m_{30} \\
& -0.2578\, m_{20}m_{30} + 0.0984\, (\tilde{m}_{D_3}^0)^2 + 0.0534\, (m_{H_1}^0)^2 - 0.0341\, (m_{H_2}^0)^2 \\
& +0.0642\, (\tilde{m}_{Q_3}^0)^2 - 0.0341\, (\tilde{m}_{U_3}^0)^2 - 0.0450\, (\tilde{m}_{L_3}^0)^2 - 0.0450\, (\tilde{m}_{E_3}^0)^2,
\end{aligned}$$

$$\Sigma_\tau = +0.0009\, A_{t0}^2 - 0.0106\, A_{b0}^2 - 0.1862\, A_{\tau0}^2 + 0.0077\, A_{t0}A_{b0}$$

$$-0.0146 \, A_{t0}A_{\tau 0} + 0.1334 \, A_{b0}A_{\tau 0} + 0.0021 \, A_{t0}m_{10} - 0.0190 \, A_{b0}m_{10}$$
$$+0.0350 \, A_{\tau 0}m_{10} + 0.0007 \, A_{t0}m_{20} - 0.0350 \, A_{b0}m_{20} + 0.0743 \, A_{\tau 0}m_{20}$$
$$-0.0397 \, A_{t0}m_{30} + 0.1288 \, A_{b0}m_{30} - 0.1090 \, A_{\tau 0}m_{30} + 0.1029 \, m_{10}^2$$
$$+0.3907 \, m_{20}^2 - 2.2649 \, m_{30}^2 - 0.0140 \, m_{10}m_{20} + 0.0160 \, m_{10}m_{30}$$
$$-0.0504 \, m_{20}m_{30} - 0.2221 \, (\tilde{m}_{D_3}^0)^2 + 0.0650 \, (m_{H_1}^0)^2 + 0.0394 \, (m_{H_2}^0)^2$$
$$-0.1827 \, (\tilde{m}_{Q_3}^0)^2 + 0.0394 \, (\tilde{m}_{U_3}^0)^2 + 0.2871 \, (\tilde{m}_{L_3}^0)^2 + 0.2871 \, (\tilde{m}_{E_3}^0)^2$$

to be compared with eqs.(25,26,27). These numbers can be now used for the calculation of the soft parameters and masses using eqs.(28) with arbitrary initial conditions for the soft terms. Being calculated for $Y_t^0 = Y_b^0 = Y_\tau^0 = 5\alpha_0$ and $\alpha_0 = 0.00329$ they are practically independent of the initial values of Yukawa couplings provided the latter are big enough.

7 Towards the Physical Masses

The values of Σ_k completely define those of the soft masses for squarks, sleptons and Higgses due to linear relations which follow from the RG equations [14] and read, after relaxing the universality assumption[2],

$$\tilde{m}_{Q3}^2 = (\tilde{m}_{Q3}^0)^2 + \frac{128f_3 + 87f_2 - 11f_1}{122} + \frac{17(\Sigma_t - \Sigma_t^0) + 20(\Sigma_b - \Sigma_b^0) - 5(\Sigma_\tau - \Sigma_\tau^0)}{122}, \quad (28)$$

$$\tilde{m}_{U3}^2 = (\tilde{m}_{U3}^0)^2 + \frac{144f_3 - 108f_2 + 144/5f_1}{122} + \frac{42(\Sigma_t - \Sigma_t^0) - 8(\Sigma_b - \Sigma_b^0) + 2(\Sigma_\tau - \Sigma_\tau^0)}{122},$$

$$\tilde{m}_{D3}^2 = (\tilde{m}_{D3}^0)^2 + \frac{112f_3 - 84f_2 + 112/5f_1}{122} + \frac{-8(\Sigma_t - \Sigma_t^0) + 48(\Sigma_b - \Sigma_b^0) - 12(\Sigma_\tau - \Sigma_\tau^0)}{122},$$

$$m_{H1}^2 = (m_{H1}^0)^2 + \frac{-240f_3 - 3f_2 - 57/5f_1}{122} + \frac{-9(\Sigma_t - \Sigma_t^0) + 54(\Sigma_b - \Sigma_b^0) + 17(\Sigma_\tau - \Sigma_\tau^0)}{122},$$

$$m_{H2}^2 = (m_{H2}^0)^2 + \frac{-272f_3 + 21f_2 - 89/5f_1}{122} + \frac{63(\Sigma_t - \Sigma_t^0) - 12(\Sigma_b - \Sigma_b^0) + 3(\Sigma_\tau - \Sigma_\tau^0)}{122},$$

$$\tilde{m}_{L3}^2 = (\tilde{m}_{L3}^0)^2 + \frac{80f_3 + 123f_2 - 103/5f_1}{122} + \frac{3(\Sigma_t - \Sigma_t^0) - 18(\Sigma_b - \Sigma_b^0) + 35(\Sigma_\tau - \Sigma_\tau^0)}{122},$$

$$\tilde{m}_{E3}^2 = (\tilde{m}_{E3}^0)^2 + \frac{160f_3 - 120f_2 + 32f_1}{122} + \frac{6(\Sigma_t - \Sigma_t^0) - 36(\Sigma_b - \Sigma_b^0) + 70(\Sigma_\tau - \Sigma_\tau^0)}{122},$$

where

$$f_i = \frac{(m_i^0)^2}{b_i} \left(1 - \frac{1}{(1 + b_i\alpha_0 t)^2} \right).$$

At this level one can already make rough qualitative statements about the physical scalar masses. We note first that, as can be seen from the above equations, the sensitivity to the initial conditions reappears partly in the running of the soft scalar masses, even in the vicinity of the IRQFP. However, this dependence remains confined in the initial values

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[2] Note that even though a trace term "$Tr(Y_{hypercharge}m^2)$" is generically present in the RGE in the non-universal case, it cancels out in eq.(28)

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of the soft masses themselves in a universal (scale independent) form, and in the initial conditions of the gaugino soft masses through the f_i's and the Σ's. [The dependence on the initial values of Yukawa couplings as well as on the A^0's and the Σ^0's, that could come from the running, remain completely screened.] The ratios giving the universal sensitivity of the running soft scalar masses to the soft scalar masses initial conditions is as follows:

	$(\tilde{m}_{Q3}^0)^2$:	$(\tilde{m}_{U3}^0)^2$:	$(\tilde{m}_{D3}^0)^2$:	$(m_{H1}^0)^2$:	$(m_{H2}^0)^2$:	$(\tilde{m}_{L3}^0)^2$:	$(\tilde{m}_{E3}^0)^2$
$(\tilde{m}_{Q3})^2$	17	-3.4	-4	-3	-3.4	1	1
$(\tilde{m}_{U3})^2$	-17	40	4	3	-21	-1	-1
$(\tilde{m}_{D3})^2$	-5	1	9.25	-4.5	1	1.5	1.5
$(m_{H1})^2$	-5	1	-6	5.67	1	-1.89	-1.89
$(m_{H2})^2$	-17	-21	4	3	19.67	-1	-1
$(\tilde{m}_{L3})^2$	5	-1	6	-5.67	-1	29	-11.67
$(\tilde{m}_{E3})^2$	5	-1	6	-5.67	-1	-11.67	8.67

These numbers are renormalization scale independent and give the trend of the relative sensitivity in the vicinity of the IRQFP.

On the other hand, the dependence on the initial soft gaugino masses is renormalization scale dependent. At the electroweak scale ($t \simeq 66$), one finds that in the soft masses of the third squark generation and of the Higgs doublets the sensitivity to $(m_3^0)^2$ remains leading (by a factor of 15 to 25) as compared to $(m_2^0)^2$. In contrast, a large cancellation occurs for the sleptons, leading to comparable sensitivities to $(m_3^0)^2$ and $(m_2^0)^2$ in $\tilde{m}_{E3}$, and even a bigger sensitivity to $(m_2^0)^2$ (by a factor of 4) in $\tilde{m}_{L3}$.

To go further to the physical scalar masses, one has to consider the behaviour of the μ parameter which enters the mixing of the left and right states. The running of this parameter has the simple form $\mu(t) \sim \mu_0 exp[\int_0^t (\alpha - Y)]$ where α, Y are generic gauge and Yukawa couplings. Thus, here too, the initial conditions for the Yukawas are screened near the IRQFP in the evolution of μ, the A_i's and Σ_i's being absent anyway. However, when the electroweak symmetry breaking (EWSB) is required to take place radiatively, the μ parameter becomes, as usual, correlated to the other parameters of the MSSM at the electroweak scale. To be specific, in the leading one-loop top/stop-bottom/sbottom approximation to the EWSB conditions, the sensitivity of μ to the initial conditions will come basically from the soft scalar masses of the Higgs doublets and scalar partners of the third quark generation [15]. As stated before, the latter dependence is dominated, on one hand by the initial conditions of the soft scalar masses, in a well determined scale independent way, and on the other hand by the (scale-dependent) m_3^0 contributions. The same dependence pattern is then taken over to the physical scalar masses. A further inclusion of the scalar τ contributions to the EWSB conditions will basically not affect this dependence pattern. Indeed, although $\tilde{m}_{L3}^2$ and $\tilde{m}_{E3}^2$ have comparable sensitivity between m_2^0 and m_3^0 at the electroweak scale, they are less sensitive to this sector altogether than to the squark soft masses. All in all our analytical results allow to draw at this stage a qualitative sensitivity hierarchy for the *physical* scalar masses:

– basically no sensitivity to Yukawa couplings initial conditions (whether unified or not), or A_i^0 initial conditions (whether universal or not),

– important sensitivity to initial conditions of the soft gaugino masses, however basically only through m_3^0, *i.e.* weak sensitivity to non-universality of this sector,

– important sensitivity to initial conditions of the soft scalar masses, however through a universal scale independent pattern.

8   Conclusion

In the present paper we have obtained general analytical forms for the solutions of the one-loop renormalization group equations in the top/bottom/τ sector of the MSSM. These solutions are valid for any value of $\tan \beta$ as well as any non-universal initial conditions for the soft SUSY breaking parameters and non-unification of the Yukawa couplings. They allow a general study of the evolution of the various parameters of the MSSM and to trace back, sector-wise, the sensitivity to initial conditions of the Yukawa couplings and the soft susy breaking parameters. We have established analytically a generic screening of non-universality, in the vicinity of the infrared quasi fixed points. In practice, this property gives the general trend of the behaviour, despite the large number of free parameters, and even when one is not very close to such a quasi fixed point. This shows that non-universality of the A parameters and gaugino soft masses, as well as Yukawa unification conditions, would basically have no influence on the squark and Higgs spectra. The main input from the gaugino sector comes from the soft gluino mass contribution (which dominates by far the other two), *i.e.* insensitive to non-universality conditions of this sector. The only substantial sensitivity to non-universality is associated to the initial conditions of the scalar soft masses, but is renormalization scale independent and well defined. A similar pattern holds for the sleptons, apart from the fact that now the contribution of the wino soft mass becomes comparable to that of the gluino, yet the overall sensitivity to the gaugino sector is much smaller than in the case of the squarks.

Detailed illustrations of the physical spectrum, including the lightest Higgs, will be given in a subsequent study.

Appendix A: Screening of initial conditions and of non-universality at the IRQFP

We give here a short proof that in the regime $Y_t^0, Y_b^0, Y_\tau^0 \to \infty$ with fixed finite ratios $Y_t^0/Y_b^0 = r_1, Y_b^0/Y_\tau^0 = r_2$ the Yukawas become insensitive both to Y_k^0 and r_1, r_2. This result would be immediate from eq.(4) by dropping 1 in the denominator were it not for the fact that the u_k's have also a non-trivial dependence on the large initial conditions. Let us rewrite eqs.(5) in the form

$$
\tilde{u}_t^{(n)} = \frac{\tilde{E}_t}{(1 + 6Y_b^0 \int \tilde{u}_b^{(n-1)})^{1/6}}
$$

$$
\tilde{u}_b^{(n)} = \frac{\tilde{E}_b}{(1 + 6Y_b^0 \int \tilde{u}_t^{(n-1)})^{1/6}(1 + 4Y_b^0 \int \tilde{u}_\tau^{(n-1)})^{1/4}}
$$

$$
\tilde{u}_\tau^{(n)} = \frac{\tilde{E}_\tau}{(1 + 6Y_b^0 \int \tilde{u}_b^{(n-1)})^{1/2}}, \tag{A.1}
$$

with

$$
\tilde{u}_k^{(0)} \equiv \tilde{E}_k, \quad (k = t, b, \tau),
$$

where the twiddled quantities are obtained from the non twiddled ones by proper rescaling with r_1 or r_2, and we indicate explicitly the order of iteration. It is now easy to show inductively that if at the nth iteration

$$
\tilde{u}_k^{(n)} \sim \frac{(\tilde{u}_k^{(n)})^{FP}}{(Y_b^0)^{p_k}} \quad \text{for} \quad Y_b^0 \to \infty, \quad \text{with } 0 < p_k < 1, \tag{A.2}
$$

where $(\tilde{u}_k^{(n)})^{FP}$ is Y_b^0 independent but r_1, r_2 dependent, then the same is true at the $(n+1)^{th}$ iteration. Furthermore, since (A.2) is obviously true for $n = 1$ as can be easily seen from (A.1) we conclude that the exact u_k's behave also like

$$
\tilde{u}_k \sim \frac{(\tilde{u}_k)^{FP}}{(Y_b^0)^{p_k}} \text{ with } 0 < p_k < 1.
$$

This means that the 1's can be legitimately dropped both in eq.(4) and eq.(5). The complete cancellation of Y_b^0, r_1 and r_2 in the final result is then obvious, leading to eqs.(17, 18).

Appendix B: Convergence of iterations for the soft terms

In this appendix we prove that the convergence of the $e_i's$ and $\xi_i's$ is automatic once that of the $u_i's$ is achieved. In particular this means that a controllable behaviour is expected whatever the initial conditions for the soft parameters may be.

Let us define

$$\mathcal{E}(t) = \begin{pmatrix} 0 & e_t(t) & 0 \\ e_b(t) & 0 & 0 \\ 0 & e_\tau(t) & 0 \end{pmatrix} \tag{B.1}$$

$$\mathcal{U}(t_1;t) = \begin{pmatrix} 0 & -\mathcal{U}_b(t_1;t) & 0 \\ -\mathcal{U}_t(t_1;t) & 0 & -\mathcal{U}_\tau(t_1;t) \\ 0 & -3\mathcal{U}_b(t_1;t) & 0 \end{pmatrix} \tag{B.2}$$

where

$$\mathcal{U}_i(t_1;t) \equiv \frac{u_i(t_1)}{1/Y_i^0 + a_i \int_0^t u_i}, \quad i = t, b, \tau \tag{B.3}$$

and $a_t = a_b = 6, a_\tau = 4$,

$$\mathcal{C}(t) = \begin{pmatrix} \frac{1}{E_t}\frac{d\tilde{E}_t}{d\eta} + A_b^0 \int_0^t \mathcal{U}_b(t_1;t) & 0 & 0 \\ 0 & \frac{1}{E_b}\frac{d\tilde{E}_b}{d\eta} + \sum_{k=t,\tau} A_k^0 \int_0^t \mathcal{U}_k(t_1;t) & 0 \\ 0 & 0 & \frac{1}{E_\tau}\frac{d\tilde{E}_\tau}{d\eta} + 3A_b^0 \int_0^t \mathcal{U}_b(t_1;t) \end{pmatrix} \tag{B.4}$$

with $C(0) = 0$. The system of integral equations for the e_i's can then be written in the matrix form

$$\mathcal{E}(t) = \mathcal{C}(t) + \int_0^t \mathcal{U}(t_1;t)\mathcal{E}(t_1)dt_1 \tag{B.5}$$

To prove the convergence of $\mathcal{E}(t)$ we the define the mapping $\mathcal{E} \to \mathcal{E}'$:

$$\mathcal{E}'(t) = \mathcal{C}(t) + \int_0^t \mathcal{U}(t_1;t)\mathcal{E}(t_1)dt_1 \tag{B.6}$$

and the norm $\| \, . \, \|$ through

$$\| M(t) \| \equiv \sup_{0 \leq t \leq T} \{\max | M_{ij}(t) |\} \tag{B.7}$$

for any matrix M in a given evolution interval $[0, T]$. One then has the inequality

$$\left| \int_0^t (\mathcal{U}\mathcal{E})_{ij} \right| \leq \left(\sum_k \int_0^t |\mathcal{U}_{ik}|\right) \| \mathcal{E} \| \tag{B.8}$$

valid for any i, j. On the other hand, one has from eqs.(B.2, B.3)

$$\sum_k \int_0^t |\mathcal{U}_{ik}| = \begin{cases} \frac{\int u_b}{1/Y_b^0 + 6\int u_b} \leq \frac{1}{6} & (i = 1) \\ \frac{\int u_t}{1/Y_t^0 + 6\int u_t} + \frac{\int u_\tau}{1/Y_\tau^0 + 4\int u_\tau} \leq \frac{5}{12} & (i = 2) \\ 3\frac{\int u_b}{1/Y_b^0 + 6\int u_b} \leq \frac{1}{2} & (i = 3) \end{cases} \tag{B.9}$$

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Combining the above inequalities (B.8, B.9) with eq.(B.6) one obtains

$$\parallel \mathcal{E}'_1 - \mathcal{E}'_2 \parallel \leq \frac{1}{2} \parallel \mathcal{E}_1 - \mathcal{E}_2 \parallel \tag{B.10}$$

that is, the mapping (B.6) is a contraction, the solution to eq.(B.5) is unique and approximated at worse with an error of $1/2^n$, after n iterations. Actually, the situation is much better than given by this upper bound error, as one can see from the numerical illustrations of section **6.2**. Finally, we note that the rational is exactly the same for the convergence of the ξ's. Indeed, apart from a different definition for $\mathcal{C}(t)$, the ξ's satisfy a matrix equation similar to (B.5) with the same $\mathcal{U}$ as the one given in (B.2).

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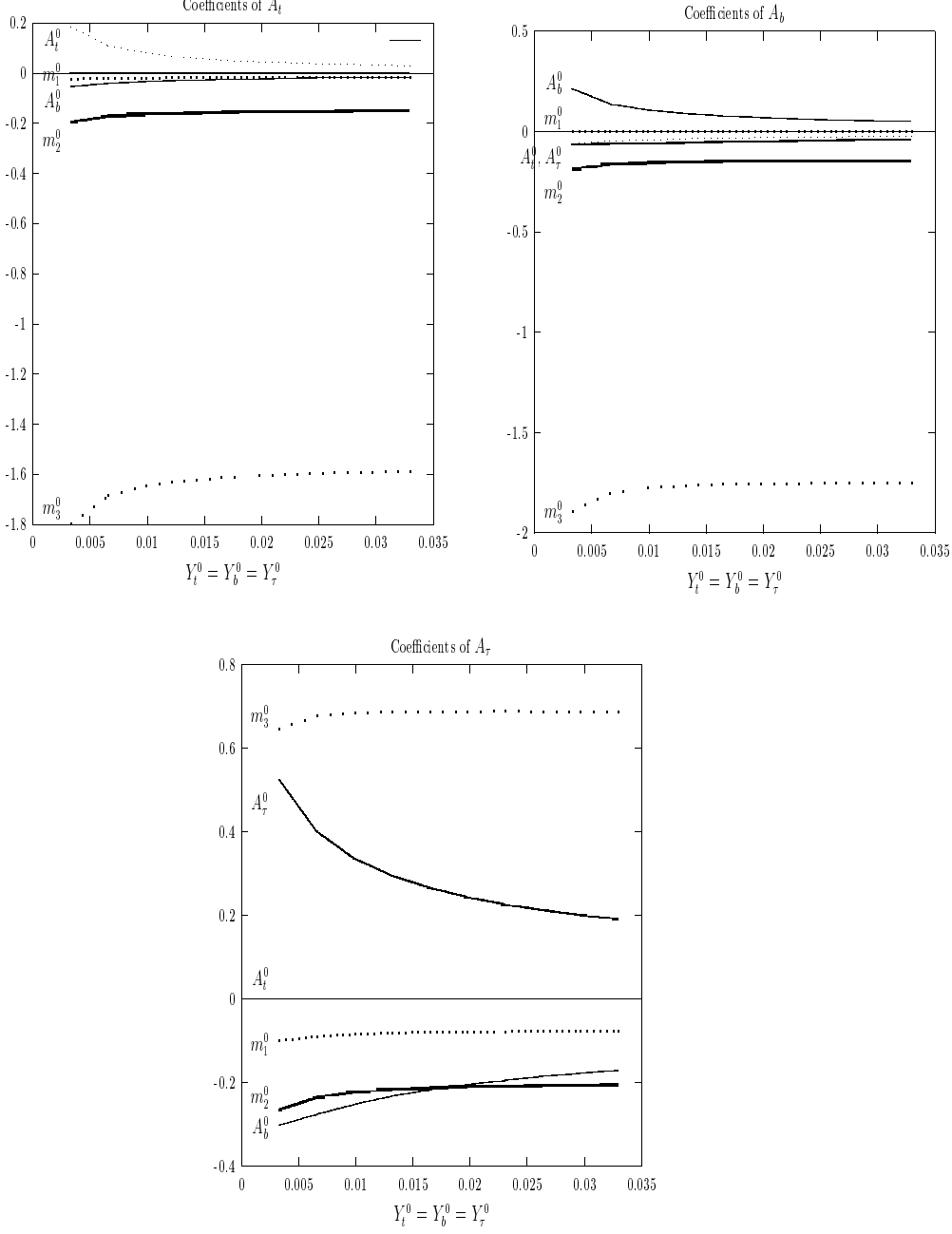
Figure 1: Initial value contributions of the various soft SUSY breaking parameters to the running A_t, A_b et A_τ at the EW scale, as a function of a common initial value for the three Yukawa couplings